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Control of a Nonlinear Aeroelastic System Using Euler–Lagrange Theory

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Active control of nonlinear aeroservoelastic (NL-ASE) systems is a key technology for future aircraft development. Because of the large number of nonlinear behaviors exhibited by NL-ASE systems, issues of local Lyapunov stability and structural stability must be addressed in controller design. A design methodology based on an ASE system being governed by the Euler–Lagrange (EL) equations is presented. The controller design procedure involves shaping the Lagrangian to a desired form and providing channels to dissipate energy. We consider EL systems with arbitrary Lagrangians and give sufficient conditions for local asymptotic stability and passivity. Parameter-dependent controllers are developed for large-scale changes in operating conditions and transitions across bifurcations. A procedure to design stabilizing controllers that are themselves Lyapunov and structurally stable is also given. The techniques are applied to a nonlinear version of benchmark active control technology wind-tunnel model that exhibits a Hopf bifurcation leading to flutter. Numerical results showing local asymptotic stability and structural stability for large-scale dynamic pressure variations across bifurcations are presented.

Nomenclature

C_L, C_M	=	lift and pitching moment coefficients
$C_{L_{(\cdot)}}$, $C_{M_{(\cdot)}}$	=	derivatives of C_L and C_M with respect to $(\cdot)$
H_p, H_c, H	=	plant, controller, and closed-loop Hamiltonians
h	=	vertical displacement (plunge) of BACT model
L_p, L_c, L	=	plant, controller, and closed-loop Lagrangians
q, Q	=	generalized displacement and forces
$\bar{q}$	=	dynamic pressure
u	=	control input
w_p	=	exogenous input
δ_{TE}, δ_{US}	=	Trailing-edge and upper spoiler control surface deflections
θ	=	pitch angle of BACT model

I. Introduction

CURRENT aircraft design practices are based largely on linear principles and have resulted in many successful applications. However, unsteady aerodynamic and nonlinear aeroservoelastic (NL-ASE) effects come to the forefront when performance requirements are made tighter, and nonlinear techniques are needed to produce successful designs. See Refs. 1–3 and the references therein for recent activities at NASA and elsewhere on nonlinear unsteady aerodynamics and NL-ASE. These research activities have identified three areas critical to the development of future aircraft: 1) aerodynamic model development, 2) nonlinear control system design, and 3) nonlinear global stability analysis (GSA). This paper is concerned primarily with control system design.

Active control of NL-ASE phenomena is a key technology for future aircraft development.[1–3] State-space techniques have been applied for local and global Lyapunov stabilization about selected equilibrium points at a fixed Mach number and/or dynamic pressure.[1,4,5] However, control strategies that guarantee structural stability, in addition to Lyapunov stability, have not been considered. Structural stability[6–8] deals with changes in local and global geometry as a function of system parameters such as dynamic pressure. This is important even in the linear case, but plays a far greater role in nonlinear systems due to the large number of phenomena that can result from structural instabilities. When large changes in operating conditions and transitions across bifurcations are required during system operation, Lyapunov stability of the closed loop at specific equilibrium points is not enough. We might consider designing a controller that is robust with respect to parameter variations. However, such a controller is highly unlikely to exist due to the large range of dynamic pressures of interest and due to the variety of behaviors encoded in NL-ASE systems. Performance of a robust controller, when it exists, may also not be acceptable. A second issue that has not been addressed is the effect of feedback on the nonlinear system away from the operating point. Stabilizing feedback will always smooth out geometry, locally producing a nice response, but may result in undesirable effects at other operating conditions. Finally, a third issue important in testing and implementation is that of controller stability. Most control theories only guarantee closed-loop stability. In this paper, we present an approach that is capable of addressing these three issues.

To introduce our control strategy, recall that an Euler–Lagrange (EL) system is a dynamical system whose equations of motion are given by[9]

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q \qquad (1)$$

where $Q \in \mathbb{R}^n$ is the generalized force and L is the Lagrangian that is a smooth scalar function of the generalized coordinate $q \in \mathbb{R}^n$ and its time derivative $\dot{q}$. EL equations given by Eq. (1) have a rich history and great philosophical significance.[9,10] Remarkably, a wide range of complex physical phenomena in a large number of disparate fields, including NL-ASE, can all be described completely by the scalar Lagrangian L and the generalized forces Q. In NL-ASE problems, the Lagrangian consists of kinetic and potential energies, whereas the generalized forces are made up of structural damping, control forces, and aerodynamic effects. These forces depend on q and $\dot{q}$. It can be shown that this dependence and global properties of the Lagrangian are the causes of catastrophic aeroelastic events.

Models of NL-ASE consist of conservative terms derived from a Lagrangian and nonconservative structural, aerodynamic, and control forces. Some basic facts about such models are worth recalling:

1) Equilibrium points of the system are related to the critical points of the Lagrangian. Under some mild conditions on the Lagrangian, all equilibrium points of a conservative system (zero nonconservative forces) are locally Lyapunov stable, but not asymptotically stable. When viewed as a map from an appropriate set of inputs to outputs, such a system is lossless.[11,12]

2) If we add a potential energy function that depends only on the generalized coordinates, then the new system with the modified Lagrangian will have a new set of equilibrium points but remains lossless.

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3) If we now introduce nonconservative Rayleigh dissipation (for example, structural damping), then at least one equilibrium point becomes locally asymptotically stable (LAS). This equilibrium point is generally the lowest potential energy state. The system is now dissipative (and passive under some additional assumptions on the Lagrangian).[11,12]

4) Aerodynamic forces and moments provide channels through which energy flows in and out of the system. This is unlike Rayleigh dissipation, which always channels energy out of the system. If the capacity of the aerodynamic supply channels is smaller than that of the dissipative channels, then the system will dissipate energy.

These observations suggest the use of control inputs to 1) shape the Lagrangian to a desired form with suitable equilibrium points and structural features and 2) provide dissipative channels with capacity larger than aerodynamic supply channels. This is the control strategy that we shall pursue in this paper.

The paper is organized as follows. In the next section, some background material on EL systems and passivity is reviewed. Analysis and controller design results are presented in Sec. III. These results extend similar results available in the literature in a number of ways and address the issue of structural stability. We also give a procedure to design controllers that are themselves EL systems. These controllers can be chosen to be Lyapunov stable and structurally stable. Application of the design techniques to a nonlinear version of benchmark active control technology (BACT) wind-tunnel model is given in Sec. IV. We shall demonstrate structural instability of the open-loop (uncontrolled) BACT model leading to flutter, structural stability of the EL-controlled BACT model, and the effect of EL controllers on closed-loop geometry. Finally, conclusions are presented in Sec. V. Because of page limitations, proofs and some nonessential results are omitted. A complete version of the paper can be obtained from the authors.

Our work in this paper will also deal with GSA, the third research area in NL-ASE. GSA involves the identification of special nonlinear phenomena such as Hopf and period doubling bifurcations, the determination of how and when such phenomena are exhibited, the determination of structural stability, and the provision of physical explanations that further our knowledge of the system. We shall perform GSA of the nonlinear BACT model with and without control in an effort to quantify the local and global effects of EL controllers. The main tool for GSA is bifurcation and chaos theory methodology (BACTM) developed[13–15] in the 1970s to study high α flight dynamics. BACTM has since found applications in fluid mechanics (Taylor vortices, Coutte flow), structural dynamics (buckling, nonlinear mechanics), population dynamics, phase transitions, chemical instabilities, compressor stall, biology, and communications. See Refs. 16 and 17 for recent applications in aircraft dynamics. BACTM works by developing a bifurcation diagram of the nonlinear system. This diagram contains location of special phenomena, loci of special solutions, and all of the information needed to determine structural stability. The numerical techniques used in these computations can be found in Refs. 13–15 and 18.

II. Background

Let us consider the EL system (1). Suppose that the Lagrangian L is smooth and can be decomposed as

$$L(q,\dot q) = T(q,\dot q) - V(q) = \tfrac{1}{2}\dot q' M(q)\dot q - V(q) \tag{2}$$

where T is the kinetic energy, V is the potential energy, and M is the mass matrix. Typically, V is bounded from below, and there exists $\epsilon > 0$ such that $M(q) > \epsilon I$ for all $q \in \mathbb{R}^n$. If the generalized forces $Q = 0$, then the total stored energy (or Hamiltonian) $H = T + V$ is a constant. When Q has the form

$$Q = u - \frac{\partial R}{\partial \dot q}$$

where u is a control input and R is the Rayleigh dissipation function, the Hamiltonian satisfies the energy balance equation

$$H(q,\dot q) - H(q_0,\dot q_0) = \int_0^t \dot q' u \,\mathrm{d}\tau - \int_0^t \dot q' \frac{\partial R}{\partial \dot q}\,\mathrm{d}\tau \tag{3}$$

which is similar to the dissipation inequalities of Willems[11,12] used in the definition of passivity. This key observation is the basis of most of the recent work on the control of EL systems (see Refs. 19–21 and the references therein for details).

In this paper, we consider EL systems with arbitrary Lagrangians as opposed to the difference between kinetic and potential energy terms[19–21] as in Eq. (2). A more general form for kinetic energy in mechanics is (see Greenwood,[22] page 263)

$$\tfrac{1}{2}\dot q' M(q)\dot q + \dot q' s(q) + t(q)$$

where s and t are smooth functions. Perhaps the most important generalization we make is regarding open-loop plant dynamics. It is usually assumed[19–21] that the plant dynamics is passive and, hence, Lyapunov stable. We shall not make this assumption in this paper. Finally, the important case of parameter-dependent EL systems is considered. The generalized forces and moments in NL-ASE depend on dynamic pressure, and the control objective becomes stabilization over a range of dynamic pressures.

III. Controller Design

Sufficient conditions for passivity and local asymptotic stability are given in this section. These conditions are then used to derive stabilizing feedback laws. The controllers given are themselves stable. We also discuss parameter-dependent EL systems and associated structural stability problems.

A. Analysis

Consider the EL system given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q} = w - \frac{\partial R}{\partial \dot q} \tag{4}$$

where q is the generalized displacement, w is a generalized exogenous input, R is a smooth Rayleigh dissipation function (of $\dot q$) satisfying

$$R(0) = 0, \qquad \dot q' \frac{\partial R}{\partial \dot q} \geq \alpha \|\dot q\|^2$$

for some $\alpha > 0$, and L is the Lagrangian that is assumed to be a smooth function of q and $\dot q$. For simplicity, we use x to denote the state of the EL system: $x = (q', \dot q')'$. Note that the function R cannot contain a linear term under the stated conditions. We say that $x_e = (q_e', 0)'$ is an equilibrium point of the EL system if

$$\frac{\partial L}{\partial q}(x_e) = 0$$

In other words, an equilibrium point is a critical point of the Lagrangian of the form $(q_e', 0)'$. Define the Hamiltonian

$$H = \dot q' \frac{\partial L}{\partial \dot q} - L \tag{5}$$

and note that $H(x_e) = -L(x_e)$ for any equilibrium point x_e. Theorem 1 gives sufficient conditions for passivity of EL systems and asymptotic stability of its equilibrium points.

Theorem 1: Consider the EL system and the associated Hamiltonian H given by Eqs. (4) and (5). Suppose that there exists an equilibrium point x_e such that

$$H(x) \geq -L(x_e) \tag{6}$$

for all x. Then the following statements are true:

1) The EL system is passive from the input w to the output $\dot q$.

2) Suppose that there exists an open neighborhood N of x_e such that x_e is the only point in N for which equality holds in Eq. (6). Then, the EL system (with $w = 0$) is LAS about x_e.

In the special case when L is given by Eq. (2), the assumption (6) in Theorem 1 means that the potential energy V is bounded from below and that the lowest potential energy state is an equilibrium point. This holds true in many problems in mechanics. In addition, if the lowest potential energy state is unique, as in statement 2, then it is LAS.

B. Memoryless Controller Synthesis

Consider a plant given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L_p}{\partial \dot q_p}\right) - \frac{\partial L_p}{\partial q_p} = w_p + B_2 u - \frac{\partial R_p}{\partial \dot q_p} + Q(q_p, \dot q_p) \tag{7}$$

where $q_p \in \mathbb{R}^{n_p}$ is the generalized coordinate of the plant, w_p is the exogenous input, u is the control input, L_p is the smooth plant Lagrangian, R_p is the Rayleigh dissipation function for the plant, and Q denotes the remaining generalized forces. This is a system with $2n_p$ states. Associated with the plant is its Hamiltonian:

$$H_p = \dot{q}_p' \frac{\partial L_p}{\partial \dot{q}_p} - L_p \tag{8}$$

We shall make the following assumptions:

Assumption 1: B_2 has full row rank, that is, $B_2 B_2' > 0$.

Assumption 2: $x_{\text{pe}} = [q_{\text{pe}}', 0]'$ is an equilibrium point of the plant, and there exists an open neighborhood $\mathcal{N}_p$ of x_{pe} such that $H_p(x) > -L_p(x_{\text{pe}})$ for all $x \in \mathcal{N}_p - \{x_{\text{pe}}\}$.

Assumption 3: For some open neighborhood $\mathcal{O}$ of q_{pe}, there is a continuous function W of $\dot{q}_p$ such that

$$\sup_{q_p \in \mathcal{O}} \dot{q}_p' Q(q_p, \dot{q}_p) \le \dot{q}_p' W(\dot{q}_p) \tag{9}$$

for all $\dot{q}_p$.

Assumption 1 means that there are at least as many independent control inputs as there are generalized coordinates (or half the total number of states). Assumption 2 is similar to the requirement in Theorem 1 of the preceding section and means that x_{pe} is a local minimum of the Hamiltonian. This is satisfied in many mechanical systems where Hamiltonian is the total energy. Assumption 3 is very important and gives a bound on the supply rate of the generalized forces. The next result gives sufficient conditions for stabilization with memoryless feedback.

Theorem 2: Consider the plant (7) with Hamiltonian (8) along with the Assumptions 1–3. Suppose that the generalized velocities $\dot{q}_p$ are available for feedback. Let

$$u = -W_1 B_2' \left(B_2 W_1^{-1} B_2'\right)^{-1} W(\dot{q}_p) \tag{10}$$

where W_1 is a smooth positive definite invertible function of $\dot{q}_p$. Then, the closed-loop system with $w_p = 0$ is LAS about x_{pe}. Moreover, if the open sets $\mathcal{N}_p$ and $\mathcal{O}$ in Assumptions 2 and 3 can be chosen to be $\mathbb{R}^{2n_p}$ and $\mathbb{R}^{n_p}$, respectively, then the closed-loop system is passive from w_p to $\dot{q}_p$.

Some remarks about the feedback law in the preceding theorem are in order. First, the open-loop plant and closed-loop system Lagrangians are the same. Moreover, the equilibrium point about which asymptotic stability can be achieved is fixed by the plant. Second, the functions W and W_1 are design variables with which requirements on performance and control saturation can be met. Third, if the open-loop plant has no Rayleigh dissipation, then we may add on a Rayleigh dissipation term to the feedback law given by Eq. (10) to guarantee LAS. Finally, the control law can be interpreted as merely trying to dissipate away energy supplied by aerodynamic channels. This is clear from the way the weight W is selected using Eq. (9) as an upper bound over the supply rate associated with the aerodynamic channels. In most NL-ASE problems, the aerodynamic forces and moments are bounded functions of states. Hence, for any $\mathcal{O}$, a weighting function W exists. Of course, a function that tightly bounds the aerodynamic contribution is preferable.

C. Dynamic Controller Synthesis

We now present a control law that can be used to shape the closed-loop Lagrangian. Consider a dynamic controller given by the EL system,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L_c}{\partial \dot{q}_c}\right) - \frac{\partial L_c}{\partial q_c} = w_c - \frac{\partial R_c}{\partial \dot{q}_c} \tag{11}$$

and the output equation,

$$u = -W_1 B_2' M^{-1}\left[W(\dot{q}_p) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L_c}{\partial \dot{q}_p}\right)\right] \tag{12}$$

where $M = B_2 W_1 B_2'$, $q_c \in \mathbb{R}^{n_c}$ is the generalized coordinate of the controller, w_c is an exogenous input, L_c is the Lagrangian that is a smooth function of $(q_c, \dot{q}_p, \dot{q}_c)$, R_c is the Rayleigh dissipation function of $\dot{q}_c$, W_1 is a positive definite invertible function, and W is as in Eq. (9). Associated with this controller is its Hamiltonian:

$$H_c = \dot{q}_c' \frac{\partial L_c}{\partial \dot{q}_c} - L_c \tag{13}$$

This controller, which we call EL controller, is an EL system with n_c generalized coordinates ($2n_c$ states). It is a simple matter to check that the closed-loop system is given by the EL system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = w - \frac{\partial R}{\partial \dot{q}} + \begin{bmatrix} Q(q_p, \dot{q}_p) - W(\dot{q}_p) \\ 0 \end{bmatrix} \tag{14}$$

where

$$q = [q_p', q_c']', \qquad w = [w_p', w_c']'$$
$$L = L_p + L_c, \qquad R = R_p + R_c \tag{15}$$

Moreover, the Hamiltonian H of the closed-loop system is the sum of the Hamiltonians of the plant and the controller. Under some mild assumptions on the controller parameters L_c and H_c along with Assumptions 1–3, we can show that the control law (11) and (12) renders the closed-loop system passive and LAS.

EL controller can be chosen to be passive, LAS, or globally asymptotically stable (GAS) by simply choosing nice Lagrangians and Hamiltonians. A particularly simple case is when $L_c = T_c - V_c$, that is, the difference of kinetic and potential energies. Because the closed-loop Lagrangian is the sum of the open-loop plant and controller Lagrangians, EL controllers can be used to remove structural instability problems.

D. Parameter-Dependent EL Systems

Consider the case where the Lagrangian L and the generalized forces Q in Eq. (4) are functions of a parameter p such as dynamic pressure. As the parameter varies, the system may exhibit a wide range of phenomena such as Hopf and period doubling bifurcations, which lead to undesirable dynamic behavior. This may happen even in the presence of a feedback law designed to stabilize about equilibrium points at a fixed value of the parameter. Our interest is in generalizing the control laws of the preceding section to guarantee structural stability across bifurcations and large variations in the parameter.

This can be accomplished in two ways. The first method is to select weighting functions that bound generalized forces over the parameter space as well. For example, in Theorem 2, choose W according to the following inequality:

$$\sup_{p \in \mathcal{P}} \sup_{q_p \in \mathcal{O}} \dot{q}_p' Q(p, q_p, \dot{q}_p) \le \dot{q}_p' W(\dot{q}_p) \tag{16}$$

for all $\dot{q}_p$. Here, $\mathcal{P}$ is a compact set to which p is known to belong and $\mathcal{O}$ is as in Assumption 3. With this weight selection, the control law given in Theorem 2 is LAS for each fixed value of parameter $p \in \mathcal{P}$. Moreover, by compactness of $\mathcal{P}$, the closed-loop system is structurally stable. Because we are using a single weight, the control effort required to stabilize may be excessive. A more reasonable method is to find a weighting function that depends smoothly on the parameter p. That is, we choose $W(p, \dot{q}_p)$ such that

$$\sup_{q_p} \dot{q}_p' Q(p, q_p, \dot{q}_p) \le \dot{q}_p' W(p, \dot{q}_p)$$

for all p and $\dot{q}_p$. This method results in a control law that is smoothly scheduled with respect to p. In a similar way, we can select parameter-dependent EL controllers for stabilization.

IV. Application to BACT Model

In this section, we first develop a nonlinear version of BACT model to test the capabilities of EL controllers. Three sets of numerical results are presented. The first case considers LAS about an open-loop unstable equilibrium point at a fixed dynamic pressure, whereas the second case deals with the effect of large initial conditions about an open-loop stable equilibrium point. In the third case, we use parameter-dependent EL controllers for stable operation over time-varying dynamic pressures. Structural stability is demonstrated in all the cases using BACTM.

A. Nonlinear BACT Model

Figure 1 shows the BACT system with its trailing-edge control surface (upper and lower spoilers are not shown). A complete description of a linear model for this setup and its experimental

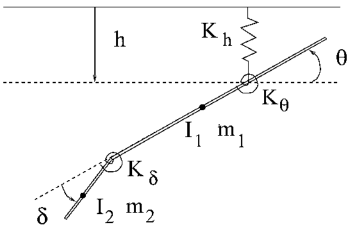

Fig. 1 BACT representation showing trailing-edge control surface.[24]

validation can be found in Refs. 23 and 24. Here, a nonlinear version that retains flutter characteristics observed in wind-tunnel test is developed.

The generalized coordinate q_p for the BACT model consists of plunge h and pitch angle θ. The kinetic and potential energies are given by

$$T_p = \tfrac{1}{2}\left[m_1(\dot{h} + e_1\dot{\theta})^2 + m_2(\dot{h} + e_2\dot{\theta} + e_\delta\dot{\delta})^2 + I_1\dot{\theta}^2 + I_2(\dot{\theta} + \dot{\delta})^2\right]$$

and

$$V_p = \tfrac{1}{2}K_h(h) + K_\theta(\theta) - m_1 g(h + e_1 \sin\theta)\cos\theta_T$$
$$- m_2 g(h + e_2 \sin\theta + e_\delta \sin\delta)\cos\theta_T$$

where $K_h(h)$ and $K_\theta(\theta)$ are strain energies. In the linear BACT model,[24] quadratic strain energy terms

$$\tfrac{1}{2}K_h h^2 + \tfrac{1}{2}K_\theta\theta^2$$

are used. Nonconservative structural forces are given by the Rayleigh dissipation function

$$R_p = [\dot{h} \quad \dot{\theta}]\begin{bmatrix} m & s_{h\theta} \\ s_{h\theta} & I_\theta \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\theta} \end{bmatrix}$$

and the external aerodynamic forces are given by the generalized force vector

$$Q(q_p, \dot{q}_p) = \bar{q}S\begin{bmatrix} -C_L \\ \bar{c}C_m \end{bmatrix}$$

where C_L and C_M are the lift and pitching moment coefficients that are approximated as linear functions of the states and control inputs. In BACT wind-tunnel tests,[23] the setup experienced a 4-Hz flutter at a dynamic pressure of approximately 148 psf at a Mach number of 0.77. The linear BACT model has a flutter frequency of 4.16 Hz occurring at 150.8 psf. Beyond 150.8 psf, the linear model is unstable.[24]

To develop a nonlinear stiffness model, we interpret the spring constants as representing equivalent linear stiffness. That is, these constants have been estimated to match observed strain energies at a maximum vertical deflection of $h_{\max}$ and maximum pitch angle of $\theta_{\max}$. We propose a strain energy model of the form

$$\tfrac{1}{2}\hat{K}_{h1}h^2 + \tfrac{1}{4}\hat{K}_{h2}h^4$$

where the constants $\hat{K}_{h1}$ and $\hat{K}_{h2}$ are determined as follows.

Assume that the contribution to the total strain energy from the fourth-order term at $h_{\max}$ is 5%. This assumption is made so that the resulting nonlinear model is not very different from the linear BACT model[24] at moderate vertical deflections. Taking $h_{\max} = 0.2$ m, this leads to $\hat{K}_{h2} = 4\hat{K}_{h1}$. Now, when we match the strain energies of the linear BACT model and the nonlinear model at $h_{\max}$, we get

$$\hat{K}_{h1} = K_h/1.08, \qquad \hat{K}_{h2} = 4K_h/1.08$$

Substituting these back into the strain energy expression, we get

$$\tfrac{1}{2}\left[(1/1.08)K_h h^2 + (f/1.08)K_h h^4\right] \tag{17}$$

as the strain energy model for plunge. Here, $f > 0$ is a stiffness factor that can be changed. Note that at $f = 4$ and $h = 0.2$ m, contribution to the total strain energy from the nonlinear modification is 5%. For values of $f < 4$, the effect of our modification is less than 5% and can be easily seen by plotting the strain energy of the linear BACT model and the nonlinear model just described. The strain energy model for pitch motion is similar. With these strain energy models, we arrive at the following nonlinear BACT model:

$$M\begin{bmatrix} \ddot{h} \\ \ddot{\theta} \end{bmatrix} + C\begin{bmatrix} \dot{h} \\ \dot{\theta} \end{bmatrix} + \frac{1}{1.08}\begin{bmatrix} k_h\left(h + 2f_h h^3\right) \\ k_\theta\left(\theta + 2f_\theta\theta^3\right) \end{bmatrix} = Q_{\text{grav}} + \bar{q}S\begin{bmatrix} C_L \\ \bar{c}C_m \end{bmatrix}$$

where the lift and moment coefficients are

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta_{\text{TE}}}}\delta_{\text{TE}} + C_{L_{\delta_{\text{US}}}}\delta_{\text{US}}$$
$$+ (\bar{c}/2U_0)\left(C_{L_{\dot{\alpha}}}\dot{\alpha} + C_{L_q}\dot{\theta}\right)$$
$$C_M = C_{M_0} + C_{M_\alpha}\alpha + C_{M_{\delta_{\text{TE}}}}\delta_{\text{TE}} + C_{M_{\delta_{\text{US}}}}\delta_{\text{US}}$$
$$+ (\bar{c}/2U_0)\left(C_{M_{\dot{\alpha}}}\dot{\alpha} + C_{M_q}\dot{\theta}\right)$$

In the preceding expressions, U_0 is the freestream velocity and α denotes the angle of attack given by

$$\alpha = \theta_T + \theta + (\dot{h}/U_0) + (l\dot{\theta}/U_0)$$

where θ_T is the turntable angle and l is the distance from the origin of the body-fixed coordinate system to the point at which α is referenced.[24] Experimentally obtained structural and aerodynamic data for the model can be found in Ref. 24. For simplicity, we shall set $f_\theta = 0$ and $f_h = 1$ in the numerical results given hereafter.

B. GSA of Open-Loop BACT Model

Figure 2 shows a bifurcation diagram of the open-loop model ($\delta_{\text{TE}} = \delta_{\text{US}} = 0$, $\theta_T = 1.6$ deg, $f_h = 1$, and $f_\theta = 0$). The bifurcation parameter is dynamic pressure $\bar{q}$. At $\bar{q} \approx 150.3$ psf, a Hopf bifurcation is detected. The equilibrium points for $\bar{q} < 150.3$ psf are LAS, whereas the equilibrium points for $\bar{q} > 150.3$ psf are unstable. A locus of periodic solutions emanating from the Hopf bifurcation (dash–dot line) is also shown.

We computed periodic solutions, associated flutter frequency (4.17 Hz) and performed limit cycle stability analysis by determining the Poincaré map and Floquet multipliers. One of the Floquet multipliers will always be on the unit disk.[7] The location of the remaining multipliers determine stability. If all of the remaining ones are inside the unit disk, then the limit cycle is stable. If at least one of them is outside the unit disk, then the limit cycle is unstable. If more than one lies on the unit disk, then further analysis is required. For the nonlinear BACT model, the Floquet multipliers were computed to be

$$(1, 1, 0.25055054 \pm j0.68005603)$$

which indicated the need for further analysis. We analyzed this situation using Hill's determinant and concluded that the limit cycle is stable, but nonhyperbolic. This conclusion can be easily verified by a numerical simulation.

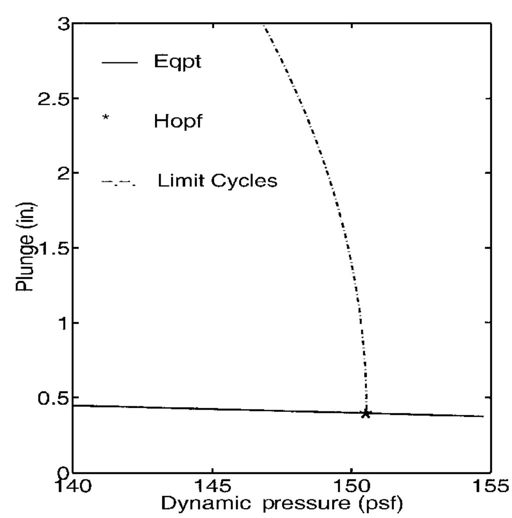

Fig. 2 Bifurcation diagram for the nonlinear BACT model.

The diagram in Fig. 2 indicates the following open-loop behavior. For each fixed $\bar{q} < 150.3$ psf, depending on the initial condition, either the motion converges to an equilibrium point or ends up in a stable limit cycle (the boundary of region of attraction of a LAS equilibrium point is made up of trajectories; our conclusion is a consequence of the locus of periodic solutions shown). That is, small initial conditions at these dynamic pressures lead to LAS behavior, whereas large initial conditions lead to limit cycling. For each fixed $\bar{q} > 150.3$ psf, motion starting at points other than the equilibrium point ends up in a limit cycle. This transition of stability properties due to the Hopf bifurcation manifests as flutter.

C. Selection of Weighting Functions in Control Law

Because the aerodynamic forces and moments used in BACT model are linear in the state variables, the weight W defined by Eq. (9) can be determined analytically. To do this, we write the aerodynamic forces and moments as

$$Q_{\text{aero}} = D_a y + K_a \theta + Q_0$$

where

$$D_a = \frac{\bar{q}S}{U_0}\begin{bmatrix} -C_{L\alpha} & -lC_{L\alpha} - (\bar{c}/2U_0)\left(C_{L\dot{\alpha}} + C_{L_q}\right) \\ \bar{c}C_{M_\alpha} & \bar{c}lC_{M_\alpha} - \left(\bar{c}^2/2U_0\right)\left(C_{M_{\dot{\alpha}}} + C_{M_q}\right)\end{bmatrix}$$

$$K_a = \bar{q}S\begin{bmatrix} -C_{L\alpha} \\ \bar{c}C_{M_\alpha}\end{bmatrix}, \qquad y = \begin{bmatrix} \dot{h} \\ \dot{\theta}\end{bmatrix}$$

and Q_0 does not depend on the generalized displacements or velocities. Given that the pitch deflection θ lies in the range $(-\theta_{\max}, \theta_{\max})$, the optimization indicated in Eq. (9) can be solved to get

$$\sup_{\theta \in (-\theta_{\max},\, \theta_{\max})} y' Q_{\text{aero}} = y' D_a y + y'(Q_0 + \theta_{\max} K_a \sigma[y' K_a])$$

where $\sigma[x]$ is the sign of x. Therefore,

$$W(y) = (D_a + \epsilon I)y + Q_0 + \theta_{\max} K_a \sigma[y' K_a] \tag{18}$$

where $\epsilon > 0$ has the required property. With this W and $W_1 = 1$, the memoryless control law becomes

$$u = -B_2'(B_2 B_2')^{-1}\left\{(D_a + \epsilon I)\begin{bmatrix}\dot{h} \\ \dot{\theta}\end{bmatrix} + Q_0 + \theta_{\max} K_a \sigma[(\dot{h} \quad \dot{\theta})K_a]\right\} \tag{19}$$

and will be used in the numerical simulations after replacing the sign function σ with a smooth approximation to avoid chatter. The scalar ϵ is to be chosen so that the closed-loop response decays satisfactorily and control inputs are within limits. We use $\epsilon = 20$ for the numerical results.

D. LAS About an Open-Loop Unstable Equilibrium Point

Small initial conditions can lead to limit cycle oscillations for all dynamic pressures $\bar{q} > \bar{q}_{\text{Hopf}} = 150.3$ psf. We shall show that EL controllers can provide LAS about an unstable equilibrium point by looking at $\bar{q} = 151$ psf. Open- and closed-loop simulation results are shown in Figs. 3 and 4. Some state-dependent process and measurement noises were used in the simulation. Actuator dynamics was not included in these simulations.

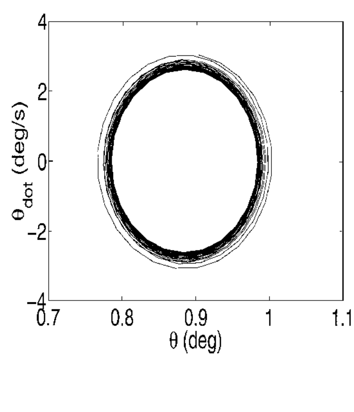

Fig. 3 Open-loop BACT response at q = 151 psf.

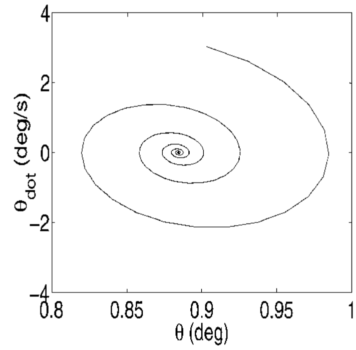

Fig. 4 Closed-loop BACT response at q = 151 psf.

As expected, the open-loop response shown in Fig. 3 starts in the vicinity of the unstable equilibrium point and ends up in a limit cycle (even if we start at the equilibrium point, process noise will drive the system into a limit cycle). The closed-loop response in Fig. 4 shows LAS about the equilibrium point. The control surface deflections δ_{TE} and δ_{US} generated by the EL controller (not shown) were small and within the limits of BACT model.

We can show using BACTM that the closed-loop system has no Hopf bifurcation or other structural instabilities in the dynamic pressure range 140–155 psf. All of the equilibrium points are LAS, and there is no change in the stability of the closed-loop system as a function of dynamic pressure. This makes a strong case for active control of NL-ASE over passive methods such as stiffening the structure, which simply postpones flutter to higher Mach numbers.

E. GAS About Equilibrium Points, Large Initial Conditions

The open-loop nonlinear BACT model is LAS for each dynamic pressure $\bar{q} < \bar{q}_{\text{Hopf}}$. However, at these dynamic pressures, large initial conditions lying outside the domain of attraction lead to limit cycle oscillations. We show that the EL controllers can be used for stabilization even in this case. For the numerical results given hereafter, we take $\bar{q} = 140 < \bar{q}_{\text{Hopf}} = 150.3$ psf. The same memoryless control law as in the earlier case was used. Closed-loop simulation results are in Figs. 5 and 6. It is clear that the response is AS. The trailing-edge control surface deflection δ_{TE} generated by the EL controller shown Fig. 6 is larger than the earlier case, but acceptable and within limits. To conclude GAS, we have to show similar response for any initial condition. An easier method is to perform

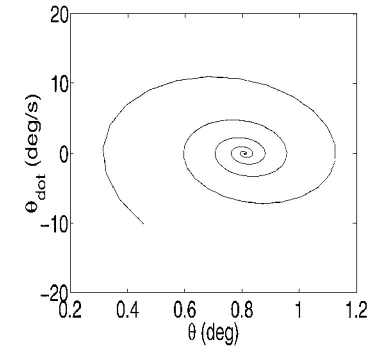

Fig. 5 Closed-loop BACT response at q = 140 psf.

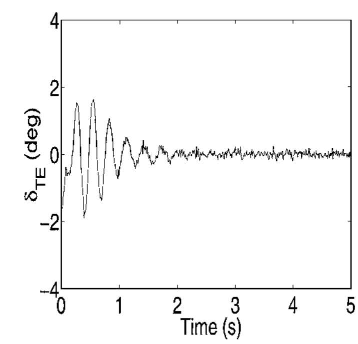

Fig. 6 Trailing-edge control deflection generated by memoryless controller (q = 140 psf).

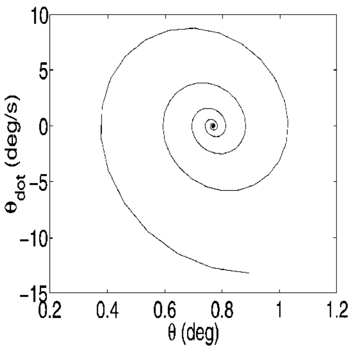

Fig. 7 Closed-loop BACT response for time-varying dynamic pressure.

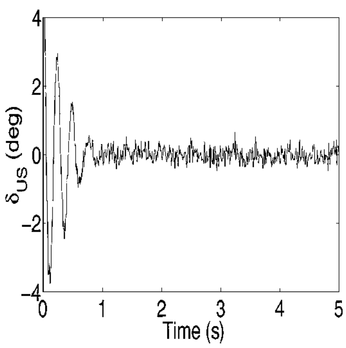

Fig. 8 Upper spoiler control deflection for time-varying dynamic pressure.

GSA using BACTM. Note that the controller being used is the same as in the preceding section. Hence, the closed-loop systems and the corresponding bifurcation diagrams are the same. We can now show that the system is GAS about the equilibrium point at $\bar{q} = 140$ psf using the following: 1) There is only equilibrium point at $\bar{q} = 140$ psf. 2) A bifurcation diagram of the closed-loop system contains no special phenomenon.

F. AS over Time-Varying Dynamic Pressures

Here, we consider stabilization of BACT model as dynamic pressure is varied in some range, for example, from $\bar{q}_{\min} = 140$ to $\bar{q}_{\max} = 155$ psf linearly in 5 s. This corresponds to stabilization in the presence of large changes in operating conditions across bifurcations. The memoryless control law is scheduled with respect to $\bar{q}$ using the relation $W = \hat{W}(\bar{q} - \bar{q}_{\min})/(\bar{q}_{\max} - \bar{q}_{\min})$ where $\hat{W}$ is the weight derived in Eq. (18). Closed-loop simulation results are shown in Figs. 7 and 8. It is clear that the response is AS.

V. Conclusions

In this paper, we outlined a control strategy for NL-ASE systems based on energy flow. Controller design procedure involves shaping the open-loop Lagrangian to a desired form and adding dissipative channels. The resulting EL controllers guarantee LAS and structural stability of the closed-loop system. We also indicated how to design EL controllers that are themselves stable. The design techniques were applied to a nonlinear version of a BACT wind-tunnel model. Numerical results demonstrate the capabilities of the control strategy including LAS and structural stability.

There are some limitations to our approach. Most important, EL controllers require generalized velocities and/or generalized displacements for feedback. In many practical problems, this may require a way to estimate states from measurements (such as the use of a Kalman filter). Also, performance of the resulting closed-loop system is dictated by the choice of weighting functions given in our theoretical results. These weighting functions can be thought of as bounds on the aerodynamic supply channels; a rough estimate of aerodynamic capacity may result in a controller with large control surface deflections and poor performance. A great deal of aerodynamic insight is required to select a good set of weights. Research is currently underway to develop control-oriented models of unsteady nonlinear aerodynamics.

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